

AUDITORY AUGMENTATION OF HAPTIC GRAPHS: DEVELOPING A GRAPHIC TOOL FOR TEACHING PRECALCULUS SKILL TO BLIND STUDENTS

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ABSTRACT

This paper discusses the development of a graphic tool to assist in the teaching of pre-calculus skills to blind students. More specifically, it reviews previous and on-going efforts to develop an instrument that will facilitate or enable blind students to examine and explore data and abstract graphs, and other mathematic entities haptically.

The paper also discusses current research plans to explore the combination of auditory and haptic stimuli to present mathematical information and concepts in a graphic, but non-visual form. In particular, much of this paper examines a number of research issues that, in the opinion of the research team, must be studied, if not resolved in order to effectively employ haptic and sonification techniques to the presentation of graphic information. In the end, the purpose of these efforts is to provide a readily useable tool that will empower blind students to learn mathematical concept in a manner comparable to sighted students.

1. INTRODUCTION

"Part of the literacy standard, then, the floor for all students, must be this: when you leave middle school, you are ready to engage with the college preparatory sequence in high school. It's a moving target, but however it's defined, it must then be seen as another floor: when you leave high school, you must be able to engage college curricula in math and science, for full college credit."

Robert P. Moses, *Radical Equations*[1]

"Every schoolchild must be educated for a productive and contributory place in an advanced information age... K through 12 is the real challenge. As a start, we begin with the assumption that all children can be educated in math and science. This may sound so elementary as to be downright silly! In some places, the educational approach is to sift and sort students early-on. This tells some students right at the starting gate that they can't master science and math -- that we do not expect them to succeed. This becomes a self-fulfilling prophecy, damning to the student and destructive for the country. We must believe in all children so that they learn to believe in themselves..."

Rita R. Colwell [2]

In West Virginia one mathematics content standard is "investigate and sketch the graphs of polynomials and rational functions using the characteristics of zeros, upper and lower

bounds, y-intercepts, symmetry, asymptotes and end behavior, maximum and minimum points and domain and range" [3].

These skills are necessary for a student to be successful in calculus and therefore to major in a STEM (science, technology, engineering, mathematics) field.

But, how can a visually impaired student meet this standard?

Some commercial products can be and are used. For example, Graph-It is a scientific graphing calculator package, which produces output which can be used to produce a Braille graph on an embosser.

However such products have a major disadvantage for classroom use in that they do too much of the work. That is, the student enters the function to be studied and receives back the graph of the function without the student doing the analytical work that is part of the requirement of the content standard.

This is a problem both in doing homework and in taking tests. How can a student learn skills without practice and how can a student demonstrate the skills expected of sighted classmates when using tools that do "too much" of the work?

Abraham Nemeth, developer of the Nemeth Code for transcribing math and science to Braille, in describing his experiences as a college student of mathematics said, "Some professors proposed giving me the test orally. I always discouraged this approach; it may convey my understanding of the basic concepts, or lack thereof, but an oral test does not reveal any skill I have in manipulating mathematical expressions. This manipulative skill is carefully nurtured and cultivated from the earliest grades, and it should pay off when performing a substantial differentiation or integration or in solving a differential equation. This manipulation cannot be done efficiently during an oral test." [4]

Graphs have long been a primary tool for conveying information and understanding about data. Notably, and perhaps most obviously, this is particularly true in the area of mathematics instruction. From number lines in elementary school to complex graphs of functions, curves and surfaces graphs are clearly central to the instruction of mathematics. This is at least the case for sighted students. However, blind students would seem to be at a significant disadvantage due their limited access to graphs through which mathematic concepts and ideas are conveyed. This has profound impact on students who are blind. Not only may it limit their educational achievement and opportunities, but it also have the effect of limiting their career choices. In particular, without the mathematics skills that might be achieved through the use of graphs and other instructional tools, blind individual may be

severely limited in their opportunities to pursue careers in scientific and engineering fields – fields in which, but for the lack of math skills, they might excel.

It would be misleading to suggest that there are no tools for presenting graphs to blind mathematics students. In fact such tools have existed for some time. Probably the common means of displaying a graph to a blind mathematics student is achieved by embossing the graph on a piece of thick paper. In this way, the student can feel the graph and get some sense about the concept upon which the graph is based. Other approaches have included wood-carvings, sandpaper and glued-string to achieve a tactile form of the graph. While these approaches have been effective within a very limited scope, they are also static and very inflexible. They do not provide the student with the opportunity to “play” with the data, which may be an invaluable part of the learning process. They also preclude the instructional use of dynamic graphs such as might be used with sighted students through on-line instructional materials.

A perusal of an instructional text for most mathematics programs would readily reveal that the instruction content is replete with graphs to convey information or concepts relevant to the program’s learning objectives. For example, a review of the Algebra 1 texts by Glencoe/McGraw-Hill [5] reveals hundreds of instances where graphs in one form or another are used to facilitate learning. The same review would also reveal that, except in rare cases, these graphs have a higher level of information density than one might suppose when one contemplates a single plotted line of a linear function, for example. In the most simple cases, for instance, a plot of linear function will often include, in addition to the plot of the function itself, axes to provide the viewer with some references in numerical space, and, perhaps, tic marks to provide the viewer with some sense of scale or quantity.

An important conclusion from the above observation is that a graph, when presented visually, is usually a complex stimulus, and includes a number of stimulus dimensions that are important to convey the idea or concept in the underlying instructional material.

It seems reasonable to assume, then, that in order to present graphs in a non-visual form, their presentation must include at least somewhat comparable stimulus complexity, lest critical information represented in the graph will fail to reach the graph’s consumer.

2. HAPTICS IN THE VEL

In November 1998 the West Virginia Virtual Environments Lab was organized with major funding from West Virginia EPSCoR. In May 1999 we purchased two PHANTOMs¹, one for use with our ImmersaDesk and one for use with a PC. Our original intended use of these PHANTOMs was in molecular dynamics simulations for pharmacy research.

That summer we built a prototype mobility-training model of downtown Morgantown which we presented at the PHANTOM Users Group in October 1999 [6].

In 2000, we designed a prototype haptic math system for use in displaying functions of one variable for touch via PHANTOM. Takamitsu Kawai joined the VEL in May and did the initial implementation. We met with a WVU staff member with acquired blindness several times for advice on the evolving user interface.

We presented a paper on this work in July 2000 at the Haptic Human Computer Interaction workshop held at the University of Glasgow [7]. This meeting was organized by

Stephen Brewster and his Glasgow Multimodal Interaction Group which is part of the Glasgow Interactive Systems Group (GIST). At that time we met with Brewster’s research group and exchanged information on our similar haptic math projects.

In 2001 we added a sonification component to haptic math. Now as a user moves the PHANTOM stylus along the groove representing the function the system plays a musical note whose pitch represents the relative numeric value of the function at that point. We produce two styles of music, one based on the western chromatic scale and one based on micropitch. The chromatic scale version may be misleading in that it displays all functions as “step functions.” However we believe that it will be easier for a user to remember the chromatic scale-based music rather than the micropitch version. We have not yet, however, done the human factors research which would confirm or refute this. In October 2001 we presented a paper at the PHANTOM Users Group describing this phase of the haptic math project [8].

VEL members are now part of a vision enhancement technology research group, involving researchers in ophthalmology and mechanical engineering as well as computer science. This group has received a planning grant from the National Science Foundation for developing an Industry/University Collaborative Research Center on this theme.

2.1. Previous research in haptic math elsewhere

Similar research has been done elsewhere.

2.1.1. Stephen Brewster - University of Glasgow

Stephen Brewster at the University of Glasgow is a leader in this type of work. His 1994 doctoral dissertation at the University of Hertfordshire was on earcons, the auditory equivalent of icons used in the user interface. He leads the MultiVis (Multimodal Visualisation for Visually Impaired People) Project, a collaboration between computer scientists and psychologists. MultiVis is part of the Glasgow Interactive Systems Group (GIST), one of the largest human computer interface research groups in the UK.

Their work has focused on (1) building haptic models from scanned images such as line graphs in a text book and (2) determining whether less expensive haptic devices can be as effective as the PHANTOM in conveying information to users [9-16].

2.1.2. Kenneth Barner - University of Delaware

In 1996-1999 Kenneth Barner at the University of Delaware developed algorithms for selectively reducing the resolution of numerical models. His group built MATLAB-based software for building haptic models for use with the PHANTOM. They also worked with building models that combined touch and sound [17].

2.2. Our niche

Our proposed work is sharply focused on teaching specific mathematical skills. Our emphases will be on the quality of the human-computer interface and on human factors studies of various approaches to teaching these concepts using haptics and sound.

3. SONIFICATION IN THE VEL

We added sonification to our haptic math project in part to assist students in remembering the shapes of functions. We have applied sonification to other problems as well.

3.1. Basketball game data

We have developed algorithms for generating music from minute by minute box scores of basketball. Each player on the studied team is assigned a unique pitch (with centers mapped to the tonic). One measure of music is generated for each minute of game time. In that measure the five pitches representing the players with the most playing time in that minute are played. The number of notes, ranging from five quarter notes to 40 thirty-second notes, indicates the relative amount by which the studied team is ahead. When that team is behind the music moves to a minor key, which has the disadvantage of removing the link between pitch and player [18].

We conducted experiments using data from several seasons: 1998 in which the team reached the Sweet Sixteen round of the NCAA tournament, 1999 which was a "rebuilding" year, 2000 in which all games were played on out-of-town courts because of an asbestos abatement problem, and 2001 another rebuilding year. We described each season to test subjects. We then played music from games which our team won. Test subjects were able to identify the season of the game more often than attributable to chance [19].

3.2. 2D images

Inspired by the work of Peter Meier and his The vOICe project [20], we have also conducted experiments in mapping 2d images to music [21,22]. We applied a grid to a black and white image and mapped each row to a pitch. Scanning the grid from left to right we played the chord (in blocked or arpeggiated form) indicated by the black cells in that column and asked the test subjects to choose the picture represented by the music. About 25% of our test subjects completed a 12 item test with no errors, but about 50% made 4 or fewer correct responses.

3.3. Materials simulations

We have also worked with faculty in physics and music composition to add sound to simulations of new materials [23, 24]. In these simulations we selected snippets of music based on local geometric properties of the arrangement of atoms with the intent of assist the physicist in quickly identifying regions in which lattice defects occurred.

4. RESEARCH QUESTIONS

Does it matter? – The fundamental assumption behind this work is that blind students are significantly disadvantaged in learning a broad array mathematics skills due to their inability to use visually presented graphs. Intuitively, this seems like a reasonable assumption since graphs typically are an integral part of mathematics instruction and they are usually printed on a page or, in recent years, displayed on a computer or calculator screen. However, this may not be true, especially for students with congenital blindness. For example, it may be the case that congenitally blind persons, or even persons who have been

blind for a long time, develop other cognitive processes that enable them to be quite proficient at understanding lines, curves, surfaces, angles and so on without "viewing" tactilely or auditorily a graph, at least in the sense that sighted people know it.

So, it seems to be an empirical question whether blind students need graphs, in haptic or auditory form in order to learn mathematic skills. As likely as it seems this issue must be tested through empirical research. Part of the VEL research agenda then, it is to study this by exposing sighted and blind individuals to simple haptic graphs and auditory graphs and haptic/auditory graphs and testing their response to related learning materials. More specifically, the plan for this study is to construct a set of simple linear function graphs in auditory form, haptic form, and combined auditory/haptic form. These graphs, then will will be presented to groups of sighted (control), sighted/blind-folded (control), individuals with acquired blindness, and congenitally blind individuals. They will then be tested to determine the extended to which they can identify properties of the graphs and to infer selected principles that the graphs portray.

Complex graphs – It seems intuitively obvious that if simple graphs of linear functions can be presented auditorily and haptically, such graphs should be readily interpretable and understandable by persons with severe visual disabilities. That is, if we hear a tone increase in pitch as we move a pointer from left to right, it seems relatively easy to interpret that $f(x)$ is increasing as x increases. Similarly, in a haptic scenario the same interpretation should result from moving a pointer from left to right and observing that the graph object (line, groove, etc.) is moving up (perhaps away from the user). It is not so clear that this ease of interpretability would carry over to more complex graphs such as multi-line graphs, non-linear functions, and so on. This is an important issue since, more often than not graphs used in mathematics instruction are more complex than simple single graphs of linear functions.

Take, for example, a graph of a positively accelerating non-linear function. Unless the function has a fairly extreme curve, would an auditory graph of that function be universally discriminable from a linear function? Perhaps not. The same might said for the graph represented haptically. Unless the haptic graph user could sense two or three points on the graph at the same time, it may be difficult to tell whether it is straight line or a curved line – and that may be the point of the graph.

This problem maybe exacerbated in haptic and auditory graphs that display more than one function at the same. This type of graph is often used to illustrate mathematical concepts such as varying slopes, and rates of change, to name just few.

The second component of the VEL research program in this area will be to explore the use of auditory and haptic stimuli to portray complex graphs. The goal of this research area will to find and implement ways in which auditory and haptic stimuli can be used in combination to facilitate the interpretability and understandability of complex graphs.

Graph metadata – A visual examination of most graphs used to present data or mathematical concepts reveals that there is considerably more information on the graph than the line or shape that may be at the heart of the graph. The additional information is data about the graph and helps to add meaning to the graph. This metadata might include axes, tick marks, legends, labels and annotations. Usually, it is this metadata that makes a graph more than an abstract line or shape in a 2D or 3D space.

How, then, does one portray the metadata pertinent to a graph in an auditory or haptic graph. This seems to be a critical issue, since without the reference information provided by the metadata, a graph may be difficult to interpret, or even unusable. The answer may lie in the ability to couple auditory and haptic stimuli in the presentation of non-visual graphs. As a simple example, a student may be able to follow a plot of a function with a haptic stylus while an audio tone may serve as a legend to tell that student which among several functions he or she is exploring.

Another important component of the VEL's auditory/haptic graph research program will be the study of graph metadata and how it may be represented in a non-visual graph display.

Learning outcomes – Of course, the ultimate goal of the VEL's auditory/haptic graph program is to effect learning outcomes. More specifically, these efforts are directed toward improving learning and, in some cases, enabling blind students to learn advanced mathematics skills. Therefore, the latter phases of this research program will focus on a careful evaluation of the auditory/haptic graph tools, as developed through research and development efforts described above, in the context of their impact on learning precalculus skills in blind students.

Toward this end, the research team will develop a set of instructional materials designed to teach a limited set of precalculus mathematics skills. These materials, then, will be used to teach these precalculus skills with and without the support of the auditory/haptic graph tools. Student participants in this study will then be tested to determine how well they understood the information presented in the auditory/haptic graph displays, how well they learned the skills taught using the instructional materials, and, after a period of time, how well they retain what they learned through these instructional activities.

5. CONCLUSIONS

Research and development efforts at the West Virginia VEL have demonstrated the efficacy of both haptic and auditory stimuli to convey data to a human in need of such information. Recent work at the lab has focused on the use of haptic stimulation to inform users about shape, slope, direction, etc. of mathematic graphs. This work, as well as work at the VEL in the area of sonification of data has suggested that perception, consumption and understanding of graphs might be enhanced, or some cases, enabled by combining haptic and auditory stimuli in the presentation of these graphs. The same experience that suggests that this approach may be fruitful also suggests that there are number of issues that are not clear about exactly how to do this with respect to displaying graphs to blind individuals. Thus there are a number of open research issues that must be explored before development efforts can proceed.

Of course, the real question that underlies this work is - can the haptic/auditory graph tool be used by blind students to learn mathematics skills more quickly, more efficiently, and more effectively? If it can, then the short term goal is to raise the level of mathematical competency in blind high school and college students. However, the ultimate goal, and hope, is that an effective and efficient non-visual means to use graphs to teach selected mathematics skills to blind students will open doors of opportunity in the fields of science and engineering, that at best, are difficult to enter.

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ⁱ A PHANTOM is a device marketed by SensAble Technologies, Inc. This device includes a stylus, shaped much like a fountain pen, which a user holds and moves within a 3D space of about 20 cm³. The PHANTOM can sense this movement and relay it to a connected computer. The PHANTOM can also "drive" the stylus into positions in 3D space. More information about the PHANTOM device may found at <http://www.sensable.com>.