Dutta, A., Campbell, K. C., & Proctor, R. W. (1994). Psychophysics and Signal Detection Theory. In Workbook for Proctor and Van Zandt Human Factors in Simple and Complex Systems, Boston: Allyn and Bacon, pp. 19-24 plus Table E.10.

Psychophysics and Signal Detection Theory

Reference Chapter: Four

Objectives: To determine perceptual limits in detection tasks and practice techniques to differentiate between observer sensitivity and response bias.

Part One

Classical Psychophysics

Psychophysics is the study of how changes in external stimulation relate to changes in sensory experience. One important question asked in psychophysical research is: How can the limits of performance be accurately measured?

One method used to answer this question is the *method of limits*. In this method, series of stimuli that increase or decrease in magnitude are presented. On ascending trials, the magnitude of the stimulus is increased slightly with each presentation. On descending trials, the magnitude of the stimulus is decreased slightly with each presentation. Equal numbers of ascending and descending trials are presented. The threshold is determined by averaging the stimulus magnitudes at the points where the subject first detects (on ascending trials) or fails to detect (on descending trials) the stimulus for each trial (i.e., take the average of the first or last detected stimulus and the last or first undetected stimulus, respectively), then averaging the values across all the trials.

Another common method is the *method of constant stimuli*. In this method, stimuli of several different magnitudes are presented repeatedly, in random order. The threshold is determined by finding the stimulus magnitude at which the subject could detect the stimulus 50% of the times it was presented. If no stimulus value is detected 50% of the time, interpolation is used to approximate the correct stimulus value.

Problem 1

The following two tables contain data from an auditory detection task. For each data set, find the absolute threshold and identify the method used to determine the threshold.

Table 1

Results of Psychophysics Experiment for Problem 1.a ("Y" denotes "Yes, stimulus was detected," "N" denotes "No, stimulus was not detected")

Stimulus Value	Series									
(in arbitrary units)	1	2	3	4	5	6	7	8	9	10
11	Y				Y					
10	Y				Y		Y			
9	Y		Y		Y		Y		Y	
8	Y	Y	Y		Y		Y		Y	
7	Y	N	Y		N	Y	Y	Y	Y	
6	N	N	Y			N	Y	N	Y	
5		N	Y	Y		N	N	N	Y	Y
4		N	N	N		N		N	N	N
3		N		N		N		N		N
2				N		N				N
1				N						N

a.

Absolute threshold: ______________________________

Method used: ___________________________________

Table 2

Results of Psychophysics Experiment for Problem 1.b

Stimulus Value (in arbitrary units)	% "Yes"	% "No"
11	100	0
10	100	0
9	96	4
8	87	13
7	70	30
6	64	36
5	52	48
4	44	56
3	20	80
2	0	100
1	0	100

b.

Absolute threshold: ______________________________

Method used: ___________________________________

c. What is the major problem with classical threshold methods?

Suggest two ways this problem can be alleviated or avoided.

1. ---

2. ---

Part Two

Signal Detection Theory

Signal detection theory is the foundation for a set of techniques used to determine the sensitivity (ability to detect a stimulus) and response bias (willingness to respond that a stimulus has been detected) of an observer.

Problem 2

The performance of two quality inspectors was monitored under different pay-off conditions. In the first session they were paid $1 for every detection of a faulty product, with no penalty for calling a flawless product faulty. In the second session they were again paid $1 for every correct detection, but were also penalized $1 for every false alarm. In the third session they were paid nothing for correct detections, but were charged $1 for every false alarm. Hit rates [Pr(reject | fault)] and false alarm rates [Pr(reject | no fault)] were computed for each session (see Table 3).

Table 3
Hit and False Alarm (F.A.) Rates for Two Subjects at Three Levels of Bias

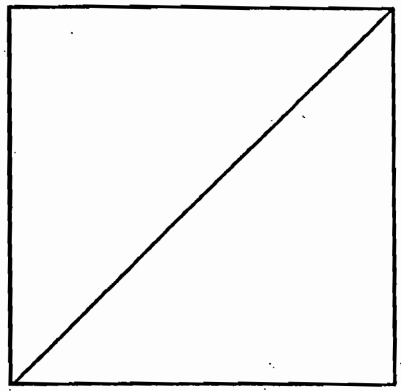
	Subject One		Subject Two	
	Hit Rate	F.A. Rate	Hit Rate	F.A. Rate
Session One:	.80	.44	.99	.63
Session Two:	.65	.27	.85	.15
Session Three:	.45	.10	.50	.02

Receiver Operating Characteristic Curves

A receiver operating characteristic (ROC) curve may be constructed for each observer by plotting hit rate on the ordinate of a square plot and false alarm rate on the abscissa. All points on a single curve have the same sensitivity (for this reason, ROC curves are sometimes called isosensitivity functions). backwards

The closer the curve comes to the point (0,1), the higher the sensitivity of the observer. Points on the curve close to (0,0) reflect a conservative bias, whereas points close to (1,1) reflect a liberal bias.

a. Plot the ROC curves for each inspector. Label the axes, the diagonal, and the ROC curves.



b. Describe inspector bias for each of the three sessions. Were both inspectors biased in the same manner?

Problem 3

The most frequently used measure of operator sensitivity is d', which is defined as the separation of the noise and signal+noise distributions. To compute d', we use the hit and false alarm rates, along with the assumption that both the noise and signal+noise distributions are normal with mean 0 and variance 1.0. The hit and false alarm rates give the *area* under the curve for the signal+noise and noise distributions, respectively. A table of areas under the standard normal curve is used to find the corresponding z score (the distance along the x axis). The formula for computing d' is:

$$d' = z(\text{Hit Rate}) + z(1.0 - \text{False Alarm Rate})$$

$$= z(\text{Hit Rate} - 0.5) + z(0.5 - \text{False Alarm Rate}),$$

where z(R) is the z score corresponding to the area R. Consider the example in Figure 1.

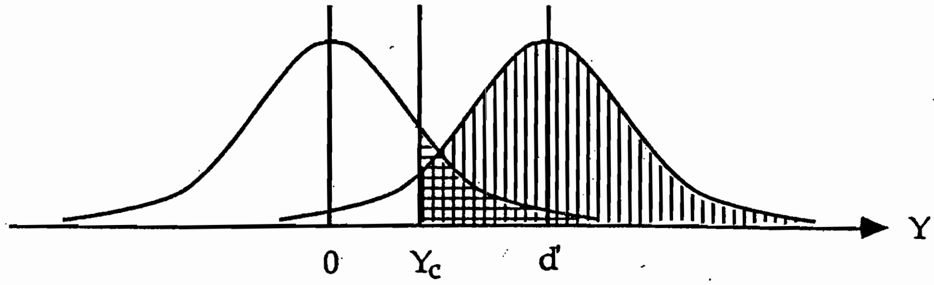


Figure 1
Noise and signal+noise distributions.

The figure shows the noise and signal+noise distributions located on the continuum of sensory evidence, Y. The mean of the noise distribution is marked by "0" and the mean of the signal+noise distribution is marked by "d'." Our goal is to find the distance between the means of the two distributions. This is the distance from 0 to Y_c (the criterion) plus the distance from Y_c to d'. The area of the noise distribution to the right of Y_c (shaded with horizontal lines) is the false alarm rate. Thus, the distance from 0 to Y_c is the z score corresponding to 0.5 (half the area under the curve) minus the false alarm rate. The area of the signal+noise distribution to the right of Y_c (shaded with vertical lines) is the hit rate. Thus, the distance from Y_c to d' is given by the hit rate minus 0.5.

a. For each pair of hit and false alarm rates given in Table 3, draw the noise and signal+noise distributions showing the means, the criterion (Y_c), and the appropriate overlap of the two distributions.

<u>Subject 1</u>

Session One	Session Two	Session Three

<u>Subject 2</u>

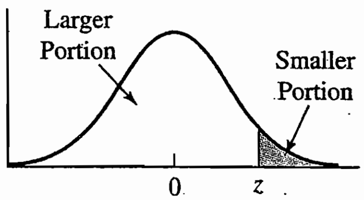
Session One	Session Two	Session Three

b. Compute d' for each pair of hit and false alarm rates given in Table 3.

	Subject One	Subject Two
Session One:	d'= _____	d'= _____
Session Two:	d'= _____	d'= _____
Session Three:	d'= _____	d'= _____

c. Does the degree of variability in the computed d's conform to your expectations? Why or why not?

Table E.10
The Normal Distribution (z)



z	Mean to z	Larger Portion	Smaller Portion	z	Mean to z	Larger Portion	Smaller Portion
.00	0.0000	0.5000	0.5000	.40	0.1554	0.6554	0.3446
.01	0.0040	0.5040	0.4960	.41	0.1591	0.6591	0.3409
.02	0.0080	0.5080	0.4920	.42	0.1628	0.6628	0.3372
.03	0.0120	0.5120	0.4880	.43	0.1664	0.6664	0.3336
.04	0.0160	0.5160	0.4840	.44	0.1700	0.6700	0.3300
.05	0.0199	0.5199	0.4801	.45	0.1736	0.6736	0.3264
.06	0.0239	0.5239	0.4761	.46	0.1772	0.6772	0.3228
.07	0.0279	0.5279	0.4721	.47	0.1808	0.6808	0.3192
.08	0.0319	0.5319	0.4681	.48	0.1844	0.6844	0.3156
.09	0.0359	0.5359	0.4641	.49	0.1879	0.6879	0.3121
.10	0.0398	0.5398	0.4602	.50	0.1915	0.6915	0.3085
.11	0.0438	0.5438	0.4562	.51	0.1950	0.6950	0.3050
.12	0.0478	0.5478	0.4522	.52	0.1985	0.6985	0.3015
.13	0.0517	0.5517	0.4483	.53	0.2019	0.7019	0.2981
.14	0.0557	0.5557	0.4443	.54	0.2054	0.7054	0.2946
.15	0.0596	0.5596	0.4404	.55	0.2088	0.7088	0.2912
.16	0.0636	0.5636	0.4364	.56	0.2123	0.7123	0.2877
.17	0.0675	0.5675	0.4325	.57	0.2157	0.7157	0.2843
.18	0.0714	0.5714	0.4286	.58	0.2190	0.7190	0.2810
.19	0.0753	0.5753	0.4247	.59	0.2224	0.7224	0.2776
.20	0.0793	0.5793	0.4207	.60	0.2257	0.7257	0.2743
.21	0.0832	0.5832	0.4168	.61	0.2291	0.7291	0.2709
.22	0.0871	0.5871	0.4129	.62	0.2324	0.7324	0.2676
.23	0.0910	0.5910	0.4090	.63	0.2357	0.7357	0.2643
.24	0.0948	0.5948	0.4052	.64	0.2389	0.7389	0.2611
.25	0.0987	0.5987	0.4013	.65	0.2422	0.7422	0.2578
.26	0.1026	0.6026	0.3974	.66	0.2454	0.7454	0.2546
.27	0.1064	0.6064	0.3936	.67	0.2486	0.7486	0.2514
.28	0.1103	0.6103	0.3897	.68	0.2517	0.7517	0.2483
.29	0.1141	0.6141	0.3859	.69	0.2549	0.7549	0.2451
.30	0.1179	0.6179	0.3821	.70	0.2580	0.7580	0.2420
.31	0.1217	0.6217	0.3783	.71	0.2611	0.7611	0.2389
.32	0.1255	0.6255	0.3745	.72	0.2642	0.7642	0.2358
.33	0.1293	0.6293	0.3707	.73	0.2673	0.7673	0.2327
.34	0.1331	0.6331	0.3669	.74	0.2704	0.7704	0.2296
.35	0.1368	0.6368	0.3632	.75	0.2734	0.7734	0.2266
.36	0.1406	0.6406	0.3594	.76	0.2764	0.7764	0.2236
.37	0.1443	0.6443	0.3557	.77	0.2794	0.7794	0.2206
.38	0.1480	0.6480	0.3520	.78	0.2823	0.7823	0.2177
.39	0.1517	0.6517	0.3483	.79	0.2852	0.7852	0.2148

(*Source:* The entries in this table were computed by the author.)

Table E.10
Continued

z	Mean to z	Larger Portion	Smaller Portion	z	Mean to z	Larger Portion	Smaller Portion
.80	0.2881	0.7881	0.2119	1.29	0.4015	0.9015	0.0985
.81	0.2910	0.7910	0.2090	1.30	0.4032	0.9032	0.0968
.82	0.2939	0.7939	0.2061	1.31	0.4049	0.9049	0.0951
.83	0.2967	0.7967	0.2033	1.32	0.4066	0.9066	0.0934
.84	0.2995	0.7995	0.2005	1.33	0.4082	0.9082	0.0918
.85	0.3023	0.8023	0.1977	1.34	0.4099	0.9099	0.0901
.86	0.3051	0.8051	0.1949	1.35	0.4115	0.9115	0.0885
.87	0.3078	0.8078	0.1922	1.36	0.4131	0.9131	0.0869
.88	0.3106	0.8106	0.1894	1.37	0.4147	0.9147	0.0853
.89	0.3133	0.8133	0.1867	1.38	0.4162	0.9162	0.0838
.90	0.3159	0.8159	0.1841	1.39	0.4177	0.9177	0.0823
.91	0.3186	0.8186	0.1814	1.40	0.4192	0.9192	0.0808
.92	0.3212	0.8212	0.1788	1.41	0.4207	0.9207	0.0793
.93	0.3238	0.8238	0.1762	1.42	0.4222	0.9222	0.0778
.94	0.3264	0.8264	0.1736	1.43	0.4236	0.9236	0.0764
.95	0.3289	0.8289	0.1711	1.44	0.4251	0.9251	0.0749
.96	0.3315	0.8315	0.1685	1.45	0.4265	0.9265	0.0735
.97	0.3340	0.8340	0.1660	1.46	0.4279	0.9279	0.0721
.98	0.3365	0.8365	0.1635	1.47	0.4292	0.9292	0.0708
.99	0.3389	0.8389	0.1611	1.48	0.4306	0.9306	0.0694
1.00	0.3413	0.8413	0.1587	1.49	0.4319	0.9319	0.0681
1.01	0.3438	0.8438	0.1562	1.50	0.4332	0.9332	0.0668
1.02	0.3461	0.8461	0.1539	1.51	0.4345	0.9345	0.0655
1.03	0.3485	0.8485	0.1515	1.52	0.4357	0.9357	0.0643
1.04	0.3508	0.8508	0.1492	1.53	0.4370	0.9370	0.0630
1.05	0.3531	0.8531	0.1469	1.54	0.4382	0.9382	0.0618
1.06	0.3554	0.8554	0.1446	1.55	0.4394	0.9394	0.0606
1.07	0.3577	0.8577	0.1423	1.56	0.4406	0.9406	0.0594
1.08	0.3599	0.8599	0.1401	1.57	0.4418	0.9418	0.0582
1.09	0.3621	0.8621	0.1379	1.58	0.4429	0.9429	0.0571
1.10	0.3643	0.8643	0.1357	1.59	0.4441	0.9441	0.0559
1.11	0.3665	0.8665	0.1335	1.60	0.4452	0.9452	0.0548
1.12	0.3686	0.8686	0.1314	1.61	0.4463	0.9463	0.0537
1.13	0.3708	0.8708	0.1292	1.62	0.4474	0.9474	0.0526
1.14	0.3729	0.8729	0.1271	1.63	0.4484	0.9484	0.0516
1.15	0.3749	0.8749	0.1251	1.64	0.4495	0.9495	0.0505
1.16	0.3770	0.8770	0.1230	1.65	0.4505	0.9505	0.0495
1.17	0.3790	0.8790	0.1210	1.66	0.4515	0.9515	0.0485
1.18	0.3810	0.8810	0.1190	1.67	0.4525	0.9525	0.0475
1.19	0.3830	0.8830	0.1170	1.68	0.4535	0.9535	0.0465
1.20	0.3849	0.8849	0.1151	1.69	0.4545	0.9545	0.0455
1.21	0.3869	0.8869	0.1131	1.70	0.4554	0.9554	0.0446
1.22	0.3888	0.8888	0.1112	1.71	0.4564	0.9564	0.0436
1.23	0.3907	0.8907	0.1093	1.72	0.4573	0.9573	0.0427
1.24	0.3925	0.8925	0.1075	1.73	0.4582	0.9582	0.0418
1.25	0.3944	0.8944	0.1056	1.74	0.4591	0.9591	0.0409
1.26	0.3962	0.8962	0.1038	1.75	0.4599	0.9599	0.0401
1.27	0.3980	0.8980	0.1020	1.76	0.4608	0.9608	0.0392
1.28	0.3997	0.8997	0.1003	1.77	0.4616	0.9616	0.0384

continued

Table E.10
Continued

z	Mean to z	Larger Portion	Smaller Portion	z	Mean to z	Larger Portion	Smaller Portion
1.78	0.4625	0.9625	0.0375	2.28	0.4887	0.9887	0.0113
1.79	0.4633	0.9633	0.0367	2.29	0.4890	0.9890	0.0110
1.80	0.4641	0.9641	0.0359	2.30	0.4893	0.9893	0.0107
1.81	0.4649	0.9649	0.0351	2.31	0.4896	0.9896	0.0104
1.82	0.4656	0.9656	0.0344	2.32	0.4898	0.9898	0.0102
1.83	0.4664	0.9664	0.0336	2.33	0.4901	0.9901	0.0099
1.84	0.4671	0.9671	0.0329	2.34	0.4904	0.9904	0.0096
1.85	0.4678	0.9678	0.0322	2.35	0.4906	0.9906	0.0094
1.86	0.4686	0.9686	0.0314	2.36	0.4909	0.9909	0.0091
1.87	0.4693	0.9693	0.0307	2.37	0.4911	0.9911	0.0089
1.88	0.4699	0.9699	0.0301	2.38	0.4913	0.9913	0.0087
1.89	0.4706	0.9706	0.0294	2.39	0.4916	0.9916	0.0084
1.90	0.4713	0.9713	0.0287	2.40	0.4918	0.9918	0.0082
1.91	0.4719	0.9719	0.0281	2.41	0.4920	0.9920	0.0080
1.92	0.4726	0.9726	0.0274	2.42	0.4922	0.9922	0.0078
1.93	0.4732	0.9732	0.0268	2.43	0.4925	0.9925	0.0075
1.94	0.4738	0.9738	0.0262	2.44	0.4927	0.9927	0.0073
1.95	0.4744	0.9744	0.0256	2.45	0.4929	0.9929	0.0071
1.96	0.4750	0.9750	0.0250	2.46	0.4931	0.9931	0.0069
1.97	0.4756	0.9756	0.0244	2.47	0.4932	0.9932	0.0068
1.98	0.4761	0.9761	0.0239	2.48	0.4934	0.9934	0.0066
1.99	0.4767	0.9767	0.0233	2.49	0.4936	0.9936	0.0064
2.00	0.4772	0.9772	0.0228	2.50	0.4938	0.9938	0.0062
2.01	0.4778	0.9778	0.0222	2.51	0.4940	0.9940	0.0060
2.02	0.4783	0.9783	0.0217	2.52	0.4941	0.9941	0.0059
2.03	0.4788	0.9788	0.0212	2.53	0.4943	0.9943	0.0057
2.04	0.4793	0.9793	0.0207	2.54	0.4945	0.9945	0.0055
2.05	0.4798	0.9798	0.0202	2.55	0.4946	0.9946	0.0054
2.06	0.4803	0.9803	0.0197	2.56	0.4948	0.9948	0.0052
2.07	0.4808	0.9808	0.0192	2.57	0.4949	0.9949	0.0051
2.08	0.4812	0.9812	0.0188	2.58	0.4951	0.9951	0.0049
2.09	0.4817	0.9817	0.0183	2.59	0.4952	0.9952	0.0048
2.10	0.4821	0.9821	0.0179	2.60	0.4953	0.9953	0.0047
2.11	0.4826	0.9826	0.0174	2.61	0.4955	0.9955	0.0045
2.12	0.4830	0.9830	0.0170	2.62	0.4956	0.9956	0.0044
2.13	0.4834	0.9834	0.0166	2.63	0.4957	0.9957	0.0043
2.14	0.4838	0.9838	0.0162	2.64	0.4959	0.9959	0.0041
2.15	0.4842	0.9842	0.0158	2.65	0.4960	0.9960	0.0040
2.16	0.4846	0.9846	0.0154	2.66	0.4961	0.9961	0.0039
2.17	0.4850	0.9850	0.0150	2.67	0.4962	0.9962	0.0038
2.18	0.4854	0.9854	0.0146	2.68	0.4963	0.9963	0.0037
2.19	0.4857	0.9857	0.0143	2.69	0.4964	0.9964	0.0036
2.20	0.4861	0.9861	0.0139	2.70	0.4965	0.9965	0.0035
2.21	0.4864	0.9864	0.0136	2.71	0.4966	0.9966	0.0034
2.22	0.4868	0.9868	0.0132	2.72	0.4967	0.9967	0.0033
2.23	0.4871	0.9871	0.0129	2.73	0.4968	0.9968	0.0032
2.24	0.4875	0.9875	0.0125	2.74	0.4969	0.9969	0.0031
2.25	0.4878	0.9878	0.0122	2.75	0.4970	0.9970	0.0030
2.26	0.4881	0.9881	0.0119	2.76	0.4971	0.9971	0.0029
2.27	0.4884	0.9884	0.0116	2.77	0.4972	0.9972	0.0028

Table E.10
Continued

z	Mean to z	Larger Portion	Smaller Portion	z	Mean to z	Larger Portion	Smaller Portion
2.78	0.4973	0.9973	0.0027	2.94	0.4984	0.9984	0.0016
2.79	0.4974	0.9974	0.0026	2.95	0.4984	0.9984	0.0016
2.80	0.4974	0.9974	0.0026	2.96	0.4985	0.9985	0.0015
2.81	0.4975	0.9975	0.0025	2.97	0.4985	0.9985	0.0015
2.82	0.4976	0.9976	0.0024	2.98	0.4986	0.9986	0.0014
2.83	0.4977	0.9977	0.0023	2.99	0.4986	0.9986	0.0014
2.84	0.4977	0.9977	0.0023	3.00	0.4987	0.9987	0.0013
2.85	0.4978	0.9978	0.0022	⋮	⋮	⋮	⋮
2.86	0.4979	0.9979	0.0021	3.25	0.4994	0.9994	0.0006
2.87	0.4979	0.9979	0.0021	⋮	⋮	⋮	⋮
2.88	0.4980	0.9980	0.0020	3.50	0.4998	0.9998	0.0002
2.89	0.4981	0.9981	0.0019	⋮	⋮	⋮	⋮
2.90	0.4981	0.9981	0.0019	3.75	0.4999	0.9999	0.0001
2.91	0.4982	0.9982	0.0018	⋮	⋮	⋮	⋮
2.92	0.4982	0.9982	0.0018	4.00	0.5000	1.0000	0.0000
2.93	0.4983	0.9983	0.0017				